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ENTRANCE EXAMINATION, 2012

**Pre-Ph.D./Ph.D.
PHYSICAL SCIENCES**

[Field of Study Code : PHYP (161)]

Time Allowed : 3 hours

Maximum Marks : 70

INSTRUCTIONS FOR CANDIDATES

- (i) This Question Paper consists of Two Parts, i.e., Part—A and Part—B.
- (ii) All questions are compulsory. Answers should be written in the space following each question.
- (iii) Use of calculators is permitted.
- (iv) Extra pages are attached at the end of the Question Paper for Rough Work.

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PART—A

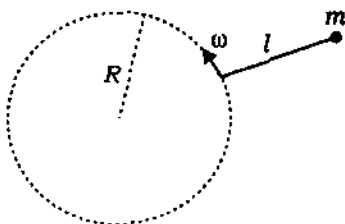
Note : Answer **all** questions. Each question carries 6 marks.

- A1.** Find the general solution to the second-order differential equation

$$(1-x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$$

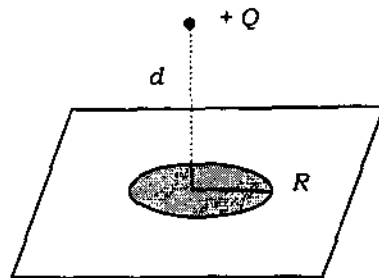
using the method of power series expansion around the point $x = 0$.

- A2.** Consider a massless rod of length l lying on a frictionless horizontal table. A particle of mass m is fixed at one end of the rod and the other end moves on a circle of radius R with a uniform angular velocity ω (see figure).



- Derive the expression for the Lagrangian of the particle.
- Derive the equation of motion of the particle.
- Show that a motion of the particle on a circular orbit is a possible solution.

- A3.** A point charge $+Q$ is placed at a distance d away from an infinite grounded conducting plane (see figure).



- What is the potential at an arbitrary point?
- Calculate the charge induced within a circle of radius R centred at a point on the plane nearest to the charge $+Q$.
- What is the force on the charge $+Q$?

- A4.** A particle of mass m moves in one dimension under the influence of a potential $V(x) = gx^4$. Estimate E_0 by variational method, starting with the wavefunction

$$\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2}$$

[Useful integrals :

$$\int_{-\infty}^{\infty} e^{-cx^2} dx = \sqrt{\frac{\pi}{c}}, \quad \int_{-\infty}^{\infty} x^2 e^{-cx^2} dx = \frac{\sqrt{\pi}}{2c^{3/2}} \quad \text{and} \quad \int_{-\infty}^{\infty} x^4 e^{-cx^2} dx = \frac{3\sqrt{\pi}}{4c^{5/2}}]$$

- A5.** At 4.2 K, the resistivity of Cu is $2.0 \times 10^{-10} \Omega\text{-m}$, the number density of Cu atoms is $8.5 \times 10^{28} \text{ m}^{-3}$ and the Fermi energy ϵ_F of the electrons is 7.0 eV.

- Calculate the Fermi velocity v_F .
- Using Drude theory, calculate the relaxation time τ_F .
- Calculate the mean free path of the electrons.

PART-B

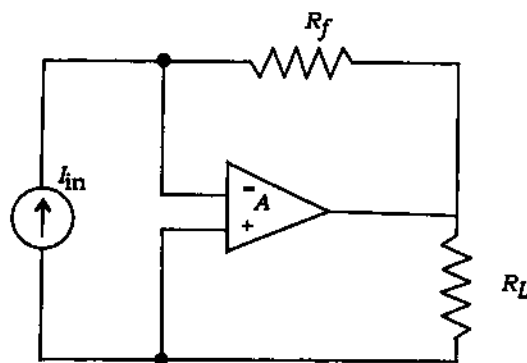
Note : Answer **all** questions. Each question carries 4 marks.

- B1.** Evaluate the integral

$$I = \frac{1}{2\pi i} \oint_{|z|=1} dz \frac{z^3}{16z^4 - 1}$$

by the method of residues.

- B2.** Consider the vector space of real-valued polynomials of degree up to 3: $V = \{a_0 + a_1x + a_2x^2 + a_3x^3 \mid \text{where } a_n \text{ are real numbers}\}$. Derive the matrix representation of the operator $\frac{d}{dx}$ in V using the functions $f_n(x) = x^{n-1}$, $n = 1, 2, 3$ and 4 as basis vectors.
- B3.** An electric dipole of moment \mathbf{p} , aligned along the positive z -direction, is placed at a point \mathbf{r} in an electric field given by $\mathbf{E} = \frac{k}{r^3} \hat{r}$, where \hat{r} denotes the unit vector in the radial direction and k is a real constant. Calculate the force experienced by the dipole.
- B4.** The Hamiltonian of a particle of mass m moving in two dimensions is given as
- $$H = \frac{1}{2m} \left(p_r^2 + \frac{1}{r^2} p_\theta^2 \right) + \frac{1}{2} m \omega^2 r^2$$
- where p_r and p_θ are the radial and angular momenta respectively, and ω is a real constant.
- (a) Write the Hamilton's equations of motion.
- (b) Identify two constants of motion for the dynamics.
- B5.** Consider the quantum state $|\Psi\rangle = |\psi_1\rangle + 2|\psi_2\rangle + 3|\psi_3\rangle$, where $|\psi_1\rangle$, $|\psi_2\rangle$ and $|\psi_3\rangle$ are normalised eigenstates of a hermitian operator H with eigenvalues 5, 2 and 4 respectively. Calculate the expectation value of H in the state $|\Psi\rangle$.
- B6.** Sodium has many isotopes of which only $^{23}_{11}\text{Na}$ is stable. In particular, the isotopes $^{22}_{11}\text{Na}$ and $^{24}_{11}\text{Na}$ are unstable under β -decay. Write the equations for the corresponding β -decay in each case.
- B7.** In the following inverting voltage feedback circuit of an operational amplifier, find the input impedance to the current source I_{in} and the output impedance to the load R_L . The open-loop gain of the op-amp is given to be A :



- B8.** Consider a system consisting of four non-interacting spin- $\frac{1}{2}$ particles, each with magnetic moment μ , in a uniform external magnetic field H . Each spin can be in one of two possible states (i) parallel to H and (ii) anti-parallel to H .
- (a) What is the total number of possible states for this system of spins?
- (b) How many of these states have total energy zero, and how many have total energy $2\mu H$?
- B9.** The Gibbs free energy of a gas is $G(T, P, N) = NT \ln\left(\frac{P}{P_0}\right) - NP a(T)$, where $a(T)$ is a function of temperature T alone and the rest of the symbols have their usual meaning with P_0 being a reference pressure. Derive the equation of state for this gas.
- B10.** For a given phonon wave vector, how many optical phonon modes are present in the NaCl crystal? Give reasons for your answer.

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PHYSICAL SCIENCES**

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Time Allowed : 3 hours

Maximum Marks : 70

INSTRUCTIONS FOR CANDIDATES

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- (ii) All questions are compulsory. Answers should be written in the space following each question.
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PART—A

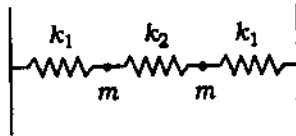
Note : Answer **all** questions. Each question carries 6 marks.

- A1.** Use the residue theorem to evaluate the integral

$$\int_{-\infty}^{+\infty} \frac{x \sin ax}{x^2 + a^2} dx$$

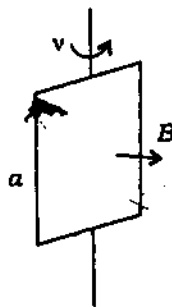
where a is a positive real number.

- A2.** Two particles, each of mass m , are connected to three springs as shown in the figure below :



The two springs of spring constant k_1 are connected to rigid walls. Considering only motion along the line joining the particles (i.e., longitudinal motion)—

- (a) write the Lagrangian of the system;
 - (b) derive the equations of motion;
 - (c) find the frequencies of the normal modes of oscillation.
- A3.** A square loop (of conducting wire) of side a and resistance R is being rotated about a vertical axis as shown in the figure. A uniform horizontal magnetic field of strength B passes through the loop. If the frequency of rotation is ν , calculate the average power dissipated in the loop.



- A4. Consider a spin-1 magnetic ion in an external magnetic field. Its Hamiltonian operator is given to be $H = \alpha S_z^2 - \beta S_x$, where α and β are positive real constants. The x- and z-components of the spin-1 matrix are

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ and } S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Find the energy eigenvalues of the ion.

- A5. Consider a system of N weakly interacting spin-1 particles. The z-component of the magnetic moment of each particle can take one of three possible values $+\mu$, 0 and $-\mu$. An external magnetic field of strength H is now applied to the system along the positive z-direction. The system is in equilibrium at temperature T .

- Calculate the magnetization of the system as a function of H and T .
- Calculate the magnetic susceptibility in the limit $H \rightarrow 0$.

PART—B

Note : Answer all questions. Each question carries 4 marks.

- B1. The action of a linear operator A on the unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} of three-dimensional space is given by $A\mathbf{i} = \mathbf{k}$, $A\mathbf{j} = \mathbf{i}$, $A\mathbf{k} = \mathbf{j}$.
- Write the matrix for A with \mathbf{i} , \mathbf{j} and \mathbf{k} as basis vectors.
 - What are the matrices for A^2 and A^3 ?
 - What are the eigenvalues of the matrix A ?

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- B2. Find the electric field \mathbf{E} everywhere in space when the electric charge density is given by

$$\rho(x, y, z) = \begin{cases} \rho_0 \left(1 - \frac{r^2}{a^2} \right) & \text{for } 0 \leq r \leq a \\ 0 & \text{for } r > a \end{cases}$$

where $r^2 = x^2 + y^2 + z^2$, and a denotes a fixed distance from the origin.

- B3. A meteorite approaches the solar system with an asymptotic speed v_0 in such a way that in the absence of any gravitational force, it would have missed the sun by a distance d . Assuming that the gravitational force on the meteorite due to any object other than the sun is negligible, what are the conservation laws applicable to the system? Use these to find the distance of closest approach of the meteorite to the sun.

- B4. Consider an electron in the state $|\ell, m\rangle$ which is a simultaneous eigenstate of L^2 and L_z .

- (a) Compute the uncertainty in the measured value of the x-component (L_x) of the orbital angular momentum.
- (b) What is its maximum value for a fixed value of ℓ ?

- B5. A set of primitive vectors for the unit cell of aluminium in a face-centred cubic lattice arrangement is given to be $\mathbf{a} = d(\mathbf{j} + \mathbf{k})$, $\mathbf{b} = d(\mathbf{i} + \mathbf{k})$ and $\mathbf{c} = d(\mathbf{i} + \mathbf{j})$. The density and mass number of aluminium are 2.7 g/cm^3 and 27 respectively. Find the value of d .

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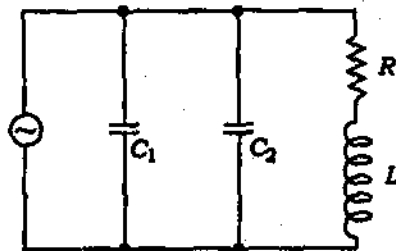
- B6. Treating the conduction electrons in copper to be free electrons, estimate the value of the Fermi energy at $T = 0$ K. Assume that each copper atom contributes one electron to conduction. The density of copper is 9 g/cm^3 and its molar weight is 63.5 g/mole .

- B7. Consider N spin- $1/2$ particles fixed at the nodes of a lattice. Their interaction is such that at very high temperatures all the spins are randomly oriented, and as $T \rightarrow 0$ all of them align in the same direction. Suppose that the heat capacity C (of the spin degrees of freedom) as a function of the temperature is given by

$$C(T) = \begin{cases} c_0(T - T_0)/T_0, & \text{for } T_0 < T < 2T_0 \\ 0, & \text{otherwise} \end{cases}$$

where c_0 and T_0 are constants. Using entropy considerations, find the value of c_0 .

- B8. Consider the following L - C oscillator circuit with $C_1 = C_2 = 15 \times 10^{-6}$ farad, $R = 50$ ohm and $L = 0.2$ henry :

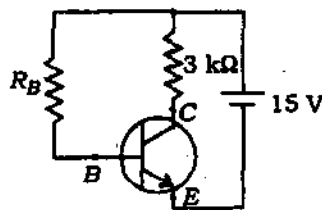


Find the frequency at resonance.

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99. Calculate the maximum ionization current generated by a source of α -rays when the α -particles are stopped completely by the gas in an ionization chamber. The energy of an α -particle from this source is 6 MeV. The source strength is 20 micro-curie (1 curie is equivalent to 3.7×10^{10} disintegrations per second) and the ionization energy for electron-ion production is 30 eV.

- B10. The value of β , the common-emitter current gain, of the silicon transistor in the following circuit is given to be 80. If the quiescent voltage $V_{CE} = 8$ V, determine the value of R_B .



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PHYSICAL SCIENCES**

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Time Allowed : 3 hours

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PART—A

Note : Answer **all** questions. Each question carries 6 marks.

- A1.** Find the general solution to the first-order differential equation

$$\frac{dx}{dt} = \lambda x(1 - x)$$

where λ is an arbitrary constant. How does $x(t)$ behave as $t \rightarrow \infty$, when $\lambda > 0$ and $\lambda < 0$ respectively?

- A2.** In addition to the planets orbiting around the Sun, there are comets in the solar system. Of these, Halley's Comet has a periodic orbit with an average period of 75.3 years.

- (a) Calculate the semi-major axis of its orbit in terms of the parameters of the Earth's orbit. (You may take the Earth's orbit to be approximately a circle, the radius of the circle is a convenient unit of distance called the *astronomical unit*, AU)
- (b) If the minimum distance of the Comet to the Sun (distance to the perihelion) is 0.6 AU, what is the eccentricity of its orbit?
- (c) What is the farthest point of the orbit from the Sun (distance to the aphelion)?

- A3.** Consider a uniform electric field \mathbf{E} along the positive z -direction and a uniform magnetic field \mathbf{B} along the positive x -direction in a right-handed coordinate system. An electrically charged particle (of mass m carrying a charge $+q$) is released with zero velocity at the origin.

- (a) Find the trajectory of the charged particle.
- (b) Make a qualitative plot of the trajectory.

- A4.** The energy levels of a two-dimensional quantum harmonic oscillator are given by $E_{n_1, n_2} = \hbar\omega (n_1 + n_2 + 1)$.

- (a) What is the degeneracy (that is, the number of quantum states) of a given energy $\hbar\omega (k + 1)$? (Here k is a fixed integer)
- (b) Evaluate the partition function $Z = \text{Tr } e^{-\beta H}$ of the system and calculate its internal energy.

- A5.** A spin-orbit interaction in an atom adds a term of the following form to the Hamiltonian of an orbiting electron :

$$H_{SO} = \mathbf{L} \cdot \mathbf{S} + \alpha(L_z + S_z)$$

where α is a real constant, \mathbf{L} and \mathbf{S} are the orbital and spin angular momentum operators, and L_z and S_z are their z -components respectively. Find all the eigenvalues of H_{SO} corresponding to $l = 1$ orbital state.

PART-B

Note : Answer all questions. Each question carries 4 marks.

- B1.** Find the conditions for a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ (where a, b, c and d are complex numbers) to be Hermitian and unitary.

- B2.** A point charge $+q$ is placed at a distance h from an infinite conducting and grounded plane. Find the induced charge density on the surface of the conducting plane.

- B3.** Let R_π be the 3×3 rotation matrix corresponding to a rotation by an angle π about any axis. Consider the matrices

$$P_\pm = \frac{1}{2}(1 \pm R_\pi)$$

Show that $P_\pm^2 = P_\pm$.

Write the explicit form of the matrix P_+ in a suitable coordinate system.

- B4.** The ground-state energy of a one-dimensional quantum harmonic oscillator is $\hbar\omega/2$. Consider a small anharmonic perturbation of the form λx^4 to the harmonic potential. Compute the correction to first order in λ to the ground-state energy. [You may want to use the relation $\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \hat{x} + \frac{i}{\sqrt{2m\omega\hbar}} \hat{p}$ between the annihilation, the position and the momentum operators.]

- B5.** The equilibrium separation between the oxygen atoms in an O_2 molecule is 1.2×10^{-10} m. Estimate the separation between the rotational energy levels corresponding to $l = 1$ and $l = 2$.

- B6.** The electric field of radiation due to a current density $\mathbf{j}(\mathbf{r}, t)$ is

$$\mathbf{E}_{\text{rad}}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \hat{\mathbf{r}} \times \left(\frac{\hat{\mathbf{r}}}{r} \times \int d^3r' \frac{\partial}{\partial t} \mathbf{j}(\mathbf{r}', t - r/c) \right)$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$ is the unit vector along \mathbf{r} . A point charge $+q$ is accelerating with its acceleration \mathbf{a} perpendicular to \mathbf{r} . Calculate the power radiated by the charge. Assume that the radiation consists of transverse electromagnetic waves with $|\mathbf{E}| = c|\mathbf{B}|$.

- B7.** A monovalent metal has a face-centred cubic (FCC) lattice structure with a lattice constant a . Show that the radius k_F of the free-electron Fermi surface is $4.90/a$.

- B8. A one-dimensional polymer is composed of N monomers, each of length a , that may be oriented along the positive or negative x -direction. Show that the force required to increase the length of the polymer by a small amount Δx at a temperature T is $-k_B T(\Delta x) / Na^2$.
- B9. Estimate the number of electrons that would be thermally excited when a metal is heated to a temperature T K. Compute the electronic heat capacity.
- B10. A relativistic neutron is travelling at half the speed of light. How much energy is required to increase its speed to $0.6c$? Compare this with the answer that you would get using non-relativistic (Newtonian) mechanics.

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